

Asymptotic Analysis of Multi-Antenna Cognitive Radio Systems Using Extreme Value Theory

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Abstract

We consider a spectrum-sharing cognitive radio system with antenna selection applied at the secondary transmitter (ST). Based on the extreme value theory, we deduce a simple and accurate expression for the asymptotic distribution of the signal to interference plus noise ratio at the secondary receiver. Using this result, the asymptotic mean capacity and the outage capacity for the secondary user (SU) are derived. The obtained asymptotic capacities approach the exact results as the number of transmit antennas N increases. Results indicate that the rate of the SU scales as $\log(N)$ when the transmit power of the ST is limited by the maximum allowable interference level, while rate scales as $\log(\log(N))$ if ST is limited by the maximum transmit power.

Index Terms

Cognitive radio, extreme value theory, mean capacity, outage capacity, rate scaling law, spectrum-sharing, transmit antenna selection.

I. INTRODUCTION

In order to utilize the radio spectrum more effectively, the secondary users (SUs) may be allowed to share the spectrum with the primary users (PUs). However, the SUs need to control the transmit power in order not to cause harmful interference to the PUs [1]. While research

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literature on this topic is rich, we recall in the following works that are most relevant for this study. Results in [2] have shown that, under a received-power constraint, the channel capacity of the SU can be significantly different from the one in a conventional AWGN channel. In [3], authors studied the ergodic capacity and the outage capacity of the SU assuming fading channels, where the interference from the PU to the SU was ignored. In [4], authors studied a single-antenna spectrum-sharing system by taking into consideration the interference from the PU at the secondary receiver (SR).

The multiple antenna techniques provide an efficient tool to improve the link level capacity. Among others, the transmit antenna selection (TAS) scheme is attractive, since it can reduce the hardware costs and complexity [5]. In [6], authors studied the ergodic capacity of the SU using TAS and maximum ratio combining (MRC) at the receiver. However, the PU interference was ignored, and no closed-form expression was given for the SU ergodic capacity in the presence of the peak transmit power and interference power constraints. Authors in [7] derived the exact expressions for the ergodic capacity of MIMO cognitive system in terms of Fox's H-function. This result, although of theoretical interests, involves complicated contour integral and is thus difficult to use in practice.

In what follows, we consider a spectrum-sharing system with multiple antennas at the secondary transmitter. By assuming the interference from the PU to the SU and the transmit power constraints at the ST, we derive the asymptotic mean capacity and outage capacity for the link between ST and SR by applying the extreme value theory (EVT) [8]. The derived results are simple but accurate, and they become exact when the number of antennas approaches to infinity. Using these results, we obtain the rate scaling laws for the spectrum-sharing systems under the interference power and the transmit power limitations.

The remainder of this paper is organized as follows. In Section II, we provide the system model and introduce the probability density function (PDF) and cumulative distribution function (CDF) of the SINR at the secondary receiver. In Section III, we briefly review the EVT, and show that the SU SINR of the proposed system lies on the domain of the attraction of the Gumbel distribution. Then we investigate the SU mean capacity, the SU outage capacity, and the scaling issues in Section IV. Section V presents the simulation results. The last Section concludes this paper.

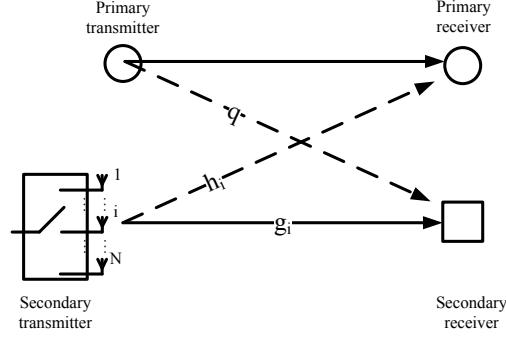


Fig. 1. Spectrum-sharing system with antenna selection at the secondary transmitter. Single antenna is equipped at the secondary receiver, the primary transmitter and receiver.

II. SYSTEM AND CHANNEL MODELS

We consider a spectrum-sharing cognitive radio system depicted in Fig. 1. This model is similar to the one in [4], where the ST was equipped with single antenna. In our model, the secondary receiver (SR) is equipped with one antenna, and the secondary transmitter (ST) is equipped with N antennas assuming TAS technique adopted at the ST. The primary transmitter (PT) and receiver (PR) are assumed to be equipped with one antenna. Let g_i , h_i ($i = 1, \dots, N$), and q denote the channel power gains between the i -th antenna of the ST and the SR, the i -th antenna of the ST and the PR, and the PT and the SR, respectively.

When the i -th antenna is used, the signal received at the SR, can be written as

$$y_i = \sqrt{P_s} g_i s + \sqrt{P_p} q z + \nu, \quad (1)$$

where P_s and P_p are the transmit power of the SU and PU, respectively, s and z denotes the transmit signals of the SU and PU with unit average power, and ν represents the additive white Gaussian noise with power σ^2 . We assume that the channels experience independent block Rayleigh fading. Namely, the channels keep constant during each block, and are uncorrelated between blocks [9]. Furthermore, the channels between the ST and the SR are identical and independent distributed (i.i.d.), and the channel between the ST and the PR is independent from the secondary channels. For a Rayleigh fading channel, the channel power gain x , $x \in \{g_i, h_i, q\}$, follows exponential distribution with mean value \bar{x} . The probability density function (PDF) of x is given by

$$f(x) = \frac{1}{\bar{x}} e^{-x/\bar{x}}, \quad x \geq 0 \quad (2)$$

and the cumulative distribution function (CDF) is

$$F(x) = 1 - e^{-x/\bar{x}}, \quad x \geq 0. \quad (3)$$

The selected antenna element of the ST provides the maximum output SINR at the SR. The SR then sends back the index to the ST through an error-free feedback channel. In order to limit the interference caused by the SU, an interference power constraint, denoted by Q , is adopted, which limits the instantaneous interference generated by the SU to the PU. In addition, P_{\max} denotes the maximum transmit power of the ST due to, for instance, hardware limitations or regulations. On limited power, now, the transmit power of the ST using the i -th antenna can be represented as [4], [10]

$$P_s^{(i)} = \min \left\{ \frac{Q}{h_i}, P_{\max} \right\}. \quad (4)$$

Accordingly, when the i -th antenna of the ST is used, the received SINR at the SR is represented as

$$\gamma_i = \min \left\{ \frac{Qg_i}{h_i(P_p q + \sigma^2)} \cdot \frac{P_{\max} g_i}{P_p q + \sigma^2} \right\} \quad (5)$$

The CDF of γ_i has been deduced in [4, eq. (15)] as

$$\begin{aligned} F_{\gamma_i}(x) = & 1 - \frac{Q\bar{g}_i}{xP_p\bar{q}\bar{h}_i} e^{\frac{\sigma^2}{P_p\bar{q}} + \frac{Q\bar{g}_i}{xP_p\bar{q}\bar{h}_i}} \\ & \times \Gamma \left[0, \left(\frac{x}{\bar{g}_i P_{\max}} + \frac{1}{P_p\bar{q}} \right) \left(\frac{Q\bar{g}_i}{x\bar{h}_i} + \sigma^2 \right) \right] \\ & - \left[1 - e^{-\frac{Q}{\bar{h}_i P_{\max}}} \right] \frac{\bar{g}_i P_{\max}}{xP_p\bar{q} + \bar{g}_i P_{\max}} e^{-\frac{x\sigma^2}{\bar{g}_i P_{\max}}}, \end{aligned} \quad (6)$$

where $\Gamma(a, b) = \int_b^\infty e^{-t} t^{a-1} dt$ denotes the incomplete Gamma function [11, eq. 8.350-2]. If the TAS technique is used, then the antenna of the ST providing the maximum SINR at the SR is selected such that

$$\gamma_{\max} = \max_i \gamma_i, \quad i = 1, \dots, N, \quad (7)$$

where γ_i is given by (5). The CDF of the received SINR at the SR can be written, using order theory [12], as

$$F_{\gamma_{\max}}(x) = [F_{\gamma_i}(x)]^N. \quad (8)$$

To the best of our knowledge, there is no explicit expressions for mean and outage capacities when using the CDF (8) due to its algebraic complexity. In order to obtain tractable analysis and gain insight for the considered cognitive radio system, we deduce asymptotic expressions for the mean capacity and outage capacity using extreme value theory.

III. IMPLICATIONS OF THE EXTREME VALUE THEORY

Let X_1, \dots, X_N denote the i.i.d. random variables, and let $F_X(x)$ be the underlying cumulative distribution function. We denote by M_X the maximum of $\{X_i\}_{1 \leq i \leq N}$. If there exist real constants a_N and $b_N > 0$, and a non-degenerate limit distribution $G(x)$ such that

$$\lim_{N \rightarrow \infty} \frac{M_X - a_N}{b_N} \xrightarrow{d} G(x), \quad (9)$$

then $G(x)$ is one of the extreme value distributions: Gumbel, Fréchet, or reverse-Weibull [13]. We say that $F_X(x)$ belongs to the maximum domain of attraction (MDA) of $G(x)$.

Before using the extreme value distributions to approximate $F_{\gamma_{\max}}(x)$ in (8), we recall a lemma proved in [8], [14] which gives a sufficient condition for a distribution belonging to the MDA of the Gumbel distribution.

Lemma 1: [8, Theorem 2.7.2] and [14, Lemma 1]. Let $\omega(F_X) = \sup \{x : F_X < 1\}$. Assume that there is a real number x_1 , such that for $x_1 \leq x < \omega(F_X)$, $f_X(x) = F'_X(x) \neq 0$ and $F''_X(x)$ exists. If

$$\lim_{x \rightarrow \omega(F_X)} \frac{d}{dx} \left[\frac{1 - F_X(x)}{f_X(x)} \right] = 0, \quad (10)$$

then $F_X(x)$ lies on the MDA of the Gumbel distribution (9) with the CDF expressed as

$$G(x) = e^{-e^{-\frac{x-a_N}{b_N}}}, \quad (11)$$

where the normalizing constants a_N and b_N are determined by

$$\begin{aligned} a_N &= F_X^{-1} \left(1 - \frac{1}{N} \right), \\ b_N &= F_X^{-1} \left(1 - \frac{1}{Ne} \right) - a_N. \end{aligned} \quad (12)$$

Here e is the base of the natural logarithm, and $F_X^{-1}(\cdot)$ denotes the inverse of the CDF.

Using Lemma 1, we prove that the CDF of SINR $F_{\gamma_i}(\cdot)$ lies on the MDA of the Gumbel distribution.

Proposition 1: Assume that the transmit power of the secondary transmitter is given by (4). Then the SINR distribution $F_{\gamma_i}(\cdot)$ of the SR with transmit antenna selection belongs to the MDA of the Gumbel distribution.

Proof: see Appendix A. ■

Using Proposition 1, we obtain approximations for the mean capacity and outage capacity of the SU.

IV. ASYMPTOTIC PERFORMANCE ANALYSIS

A. SU Mean Capacity

The instantaneous rate for the SU can be represented as

$$R_{\max} = \max_{1 \leq i \leq N} R_i, \quad (13)$$

where $R_i = \log(1 + \gamma_i)$, $\forall i = 1, \dots, N$ is the instantaneous rate using the i -th antenna. According to the limiting throughput distribution (LTD) theorem proved in [14], the distribution of R_i belongs to the MDA of the Gumbel distribution. Consequently, we have $\lim_{N \rightarrow \infty} \frac{R_{\max} - a_N}{b_N} \xrightarrow{d} G(x)$ and $G(x)$ is expressed in (11) with the following normalizing constants

$$\begin{aligned} a_N &= \log \left[1 + F_{\gamma_i}^{-1} \left(1 - \frac{1}{N} \right) \right], \\ b_N &= \log \left[\frac{1 + F_{\gamma_i}^{-1} \left(1 - \frac{1}{Ne} \right)}{1 + F_{\gamma_i}^{-1} \left(1 - \frac{1}{N} \right)} \right], \end{aligned} \quad (14)$$

where $F_{\gamma_i}^{-1}(\cdot)$ denotes the inverse function of $F_{\gamma_i}(\cdot)$ in (6). Based on the Lemma 2 in [14], we find that the expectation of $\frac{R_{\max} - a_N}{b_N}$ converges, and the mean capacity of the SU $C_s = \mathbb{E}[R_{\max}]$ can be thus approximated by integrating R_{\max} over (11) as

$$C_s \approx a_N + b_N E_0, \quad (15)$$

where $E_0 = 0.5772 \dots$ denotes the Euler constant. According to Proposition 1 the approximation is tight in the asymptotic regime $N \rightarrow \infty$.

Based on (15), we study the behavior of the SU mean capacity in two limiting regimes. First, we consider that the maximum transmit power constraint approaches infinity ($P_{\max} \rightarrow \infty$). And the second case is that the interference power constraint is much larger than P_{\max} , i.e., $Q \gg P_{\max}$. We name these two scenarios as interference power limited regime (IPLR) and transmit power limited regime (TPLR), respectively.

1) *Interference power limited regime:* When $P_{\max} \rightarrow \infty$ and the i -th antenna is selected, we rewrite the asymptotic CDF (6) of the SINR at the SR as

$$F_{\gamma_i, \text{IPLR}}(x) = 1 - c_p \frac{Q\bar{g}}{x\bar{h}}, \quad (16)$$

where $c_p = e^{\sigma^2/(P_p \bar{q})} \Gamma\left(0, \frac{\sigma^2}{P_p \bar{q}}\right) / P_p \bar{q}$ represents the effects of the additive noise and the PU interference. The inverse CDF of the SINR is given by

$$F_{\gamma_i, \text{IPLR}}^{-1}[y] = c_p \frac{Q\bar{g}}{(1-y)\bar{h}}. \quad (17)$$

Using (14) and (17), the normalizing constants of the Gumbel distribution $G(x)$ are obtained as

$$a_N^{\text{IPLR}} = \log \left(1 + c_p \frac{Q\bar{g}}{h} N \right), \quad (18)$$

and

$$b_N^{\text{IPLR}} = \log \left(\frac{1 + c_p \frac{Q\bar{g}}{h} N e}{1 + c_p \frac{Q\bar{g}}{h} N} \right), \quad (19)$$

where $b_N^{\text{IPLR}} \rightarrow 1$ as $N \rightarrow \infty$. Thus, the asymptotic mean capacity (15) becomes

$$C^{\text{IPLR}} = \log \left(1 + c_p \frac{Q\bar{g}}{h} N \right) + E_0, \quad (20)$$

which indicates that the SU mean capacity scales as $\log(N)$ in the interference power limited regime.

2) *Transmit power limited regime:* When $Q \gg P_{\max}$ and the i -th antenna is selected, we rewrite the asymptotic CDF (6) of the SINR at the SR as

$$F_{\gamma_i, \text{TPLR}}(x) = 1 - \frac{c_q P_{\max} \bar{g}}{x P_p \bar{q} + P_{\max} \bar{g}} e^{-\frac{x \sigma^2}{P_{\max} \bar{g}}}, \quad (21)$$

where $c_q = 1 - e^{-\rho/\bar{h}}$ and $\rho = Q/P_{\max}$. The inverse CDF of the SINR can be obtained as

$$F_{\gamma_i, \text{TPLR}}^{-1}[y] \approx \frac{P_{\max} \bar{g}}{\sigma^2} \mathcal{W} \left(\frac{c_q e^{\frac{\sigma^2}{P_p \bar{q}}}}{(1-y) P_p \bar{q} / \sigma^2} \right) - \frac{P_{\max} \bar{g}}{P_p \bar{q}}, \quad (22)$$

where $\mathcal{W}(x)$ denotes the Lambert W-Function satisfying $\mathcal{W}(x)e^{\mathcal{W}(x)} = x$. Thus, using (14) and (22), the normalizing constants of the Gumbel distribution $G(x)$ are obtained as

$$a_N^{\text{TPLR}} = \log \left(1 + \frac{P_{\max} \bar{g}}{\sigma^2} \mathcal{W} \left(\frac{N c_q e^{\frac{\sigma^2}{P_p \bar{q}}}}{P_p \bar{q} / \sigma^2} \right) - \frac{P_{\max} \bar{g}}{P_p \bar{q}} \right), \quad (23)$$

$$b_N^{\text{TPLR}} = \log \left(1 + \frac{P_{\max} \bar{g}}{\sigma^2} \mathcal{W} \left(\frac{e N c_q e^{\frac{\sigma^2}{P_p \bar{q}}}}{P_p \bar{q} / \sigma^2} \right) - \frac{P_{\max} \bar{g}}{P_p \bar{q}} \right) - a_N^{\text{TPLR}}, \quad (24)$$

where $b_N^{\text{TPLR}} = O(1/N)$ as $N \rightarrow \infty$, and $O(z)^m$ represents a term of order $(z)^m$. Therefore, the SU asymptotic mean capacity (15) becomes

$$C^{\text{TPLR}} = a_N^{\text{TPLR}} + b_N^{\text{TPLR}} E_0, \quad (25)$$

which admits an explicit expression and avoids the operation of inverting the SINR CDF function (6).

When the interference from the PU can be ignored, i.e. $P_p \bar{q} \rightarrow 0$, (21) becomes

$$F_{\gamma_i, \text{TPLR}}(x) \approx 1 - c_q e^{-\frac{x \sigma^2}{P_{\max} \bar{g}}}, \quad (26)$$

which is the approximation of the CDF of the SINR at low interference regime. Then we obtain the inverse CDF as

$$F_{\gamma_i, \text{TPLR}}^{-1}[y] \approx \frac{P_{\max} \bar{g}}{\sigma^2} \log \left(\frac{c_q}{1-y} \right). \quad (27)$$

Substituting (27) into (14), we have the normalizing constants

$$a_{N, \text{low}}^{\text{TPLR}} = \log \left(1 + \frac{P_{\max} \bar{g}}{\sigma^2} \log(c_q N) \right), \quad (28)$$

$$b_{N, \text{low}}^{\text{TPLR}} = \log \left(1 + \frac{P_{\max} \bar{g} / \sigma^2}{1 + \frac{P_{\max} \bar{g}}{\sigma^2} \log(c_q N)} \right). \quad (29)$$

Substituting (28) and (29) into (15), the asymptotic mean capacity is obtained as

$$C_{\text{low}}^{\text{TPLR}} \approx a_{N, \text{low}}^{\text{TPLR}} + b_{N, \text{low}}^{\text{TPLR}} E_0, \quad (30)$$

where $b_{N, \text{low}}^{\text{TPLR}} = O(1/N)$ as $N \rightarrow \infty$. This shows that the SU mean capacity scales as $\log \log(N)$ in the transmit power limited regime with low PU interference. Note that in the TPLR with low PU interference, the considered model (1) reduces to the conventional point-to-point communication. In this case, the derived result (30) is in line with [15, eq. (38)].

Remarks: The results suggest that the TAS provides different power gains in these two scenarios. In the TPLR, (30) shows that increasing the number of transmit antennas may not be an efficient way to achieve higher mean capacity, while it is effective in the IPLR as shown in (20).

B. SU Outage Capacity

Given the rate r (nats/s/Hz), the SU outage probability after transmit antenna selection can be defined as

$$F_{\text{out}}(r) = \Pr \{ R_{\max} < r \}, \quad (31)$$

where R_{\max} is given by (13). The outage capacity can be interpreted as the maximum rate such that $F_{\text{out}} \leq \epsilon$, where ϵ is the defined outage threshold. For monotonically increasing function $F_{\text{out}}(r)$, we have

$$C_{\text{out}} = F_{\text{out}}^{-1}(\epsilon). \quad (32)$$

According to the LTD theorem [14], the asymptotic distribution of R_{\max} can be approximated as (11). Thereafter, we have the following proposition for the SU outage capacity.

Proposition 2: With TAS, the asymptotic outage capacity of the SU is given by

$$C_{\text{out}} \approx a_N - b_N \log \log \left(\frac{1}{\epsilon} \right), \quad (33)$$

where the normalizing constants a_N and b_N are the normalizing coefficients given in (14).

Proof: The proof of this proposition is straightforward by substituting (11) into (32). ■

Based on Proposition 2, we investigate the SU outage capacity scaling properties in case of IPLR and TPLR, respectively.

1) *Interference power limited regime:* When $P_{\max} \rightarrow \infty$, the normalizing constants a_N^{IPLR} and b_N^{IPLR} are computed in (18) and (19), respectively. Following (33), the asymptotic SU outage capacity in IPLR yields

$$C_{\text{out}}^{\text{IPLR}} = \log \left(1 + c_p \frac{Q\bar{g}}{h} N \right) - \log \left(\frac{1 + c_p \frac{Q\bar{g}}{h} N e}{1 + c_p \frac{Q\bar{g}}{h} N} \right) \log \log \left(\frac{1}{\epsilon} \right). \quad (34)$$

Recall that $b_N^{\text{IPLR}} \rightarrow 1$ when $N \rightarrow \infty$, we have

$$C_{\text{out}}^{\text{IPLR}} \approx \log \left(1 + c_p \frac{Q\bar{g}}{h} N \right) - \log \log \left(\frac{1}{\epsilon} \right) \quad (35)$$

$$= C^{\text{IPLR}} - E_0 - \log \log \left(\frac{1}{\epsilon} \right), \quad (36)$$

where C^{IPLR} is given in (20). We can see that the SU outage capacity scales in the same manner as the SU mean capacity. When given the outage threshold ϵ , there exists a constant gap $E_0 + \log \log \left(\frac{1}{\epsilon} \right)$ between $C_{\text{out}}^{\text{IPLR}}$ and C^{IPLR} .

2) *Transmit power limited regime:* When $Q \gg P_{\max}$, following (33) the asymptotic SU outage capacity in TPLR yields

$$C_{\text{out}}^{\text{TPLR}} = a_N^{\text{TPLR}} - b_N^{\text{TPLR}} \log \log \left(\frac{1}{\epsilon} \right), \quad (37)$$

where the normalizing constants a_N^{TPLR} and b_N^{TPLR} are computed in (23) and (24), respectively. We can see that the SU outage capacity scales the same way as the SU mean capacity, i.e. $C_{\text{out}}^{\text{TPLR}} \sim \log \log (N)$ as $N \rightarrow \infty$. It is also shown that the asymptotic SU outage capacity does not depend on the outage threshold ϵ . However, the convergence speed is pretty slow due to $C_{\text{out}}^{\text{TPLR}} \sim \log \log (N)$.

When the interference from the PU can be ignored in TPLR, i.e. $P_p \bar{q} \rightarrow 0$, the asymptotic SU outage capacity (37) yields

$$C_{\text{out, low}}^{\text{TPLR}} \approx a_{N, \text{low}}^{\text{TPLR}} - b_{N, \text{low}}^{\text{TPLR}} \log \log \left(\frac{1}{\epsilon} \right). \quad (38)$$

where $a_{N, \text{low}}^{\text{TPLR}}$ and $b_{N, \text{low}}^{\text{TPLR}}$ are given by (28) and (29), respectively.

Remarks: The number of antennas has different influence on the asymptotic outage capacity of the SU. In the TPLR case, i.e. $P_{\max} \ll Q$, the SU outage capacity increases logarithmically with N and the gap between the SU mean capacity and outage capacity vanishes. However, this is not for the IPLR scenario, i.e. $P_{\max} \rightarrow \infty$, that the outage capacity has a much slower scaling rate, $\log \log(N)$. In addition, there is always a gap between the outage capacity $C_{\text{out}}^{\text{IPLR}}$ and mean capacity C^{IPLR} .

V. SIMULATION RESULTS

We present the asymptotic results with respect to N for the SU mean capacity and the outage capacity, assuming that the ST employs transmit antenna selection and the transmit power is restricted by the peak interference power constraint and the maximum transmit power constraint. The rate scaling behaviors of the mean capacity and outage capacity are also discussed. The mean channel power gains are set to 1. The noise power per Hz at the SR is set to -10 dB, i.e., $\sigma^2 = 0.1$, where we assume that the maximum average received signal to noise ratio at the SR is greater or equal to 1, i.e., $\mathbb{E}[P_{\max} \bar{g}_i] / \sigma^2 \geq 1$. Let $\text{INR} = P_p / \sigma^2$ denote the interference to the noise ratio, $\text{SIR}_Q = Q / P_p$ and $\text{SIR}_P = P_{\max} / P_p$ be the transmit signal to the interference power ratio, and $\text{SNR}_Q = Q / \sigma^2$ and $\text{SNR}_P = P_{\max} / \sigma^2$ be the transmit signal to the noise power ratio. In addition, $\text{PQR} = P_{\max} / Q$ is the maximum power to the interference constraint ratio.

Fig. 2 shows the SU mean capacity versus the interference power constraint Q when the number of the ST antennas is $N = 4, 10$, and 20 . The PU transmit power is set to $P_p = 0$ dB, and the maximum transmit power P_{\max} of the SU is 0 dB. We also plotted the asymptotic mean capacity of the SU according to (15), and compared it to the simulation results generated by 10^6 channel realizations. As the interference power constraint Q increases, the SU mean capacity is improved until the maximum transmit power constraint P_{\max} becomes dominating. We can see that the asymptotic results using EVT are reasonably accurate, which validates our approach to characterize the SU mean capacity using simple and explicit expressions.

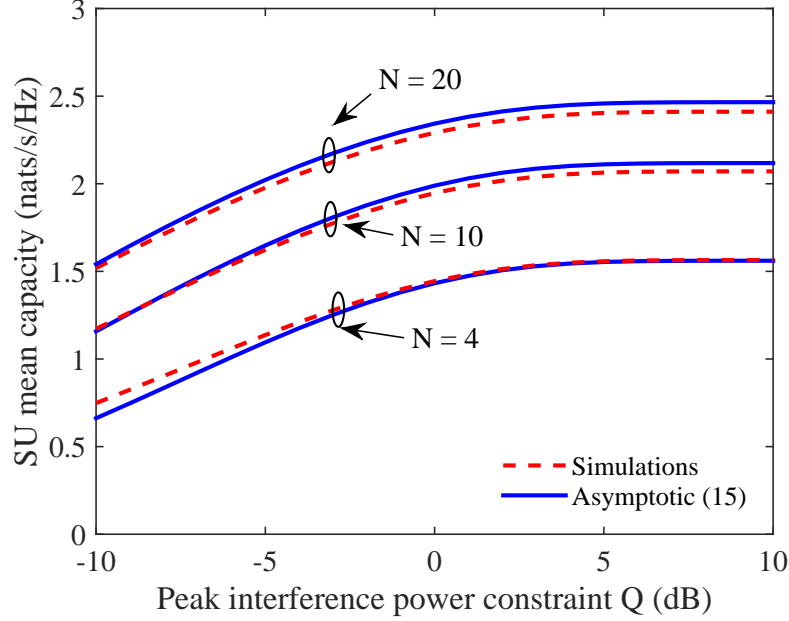
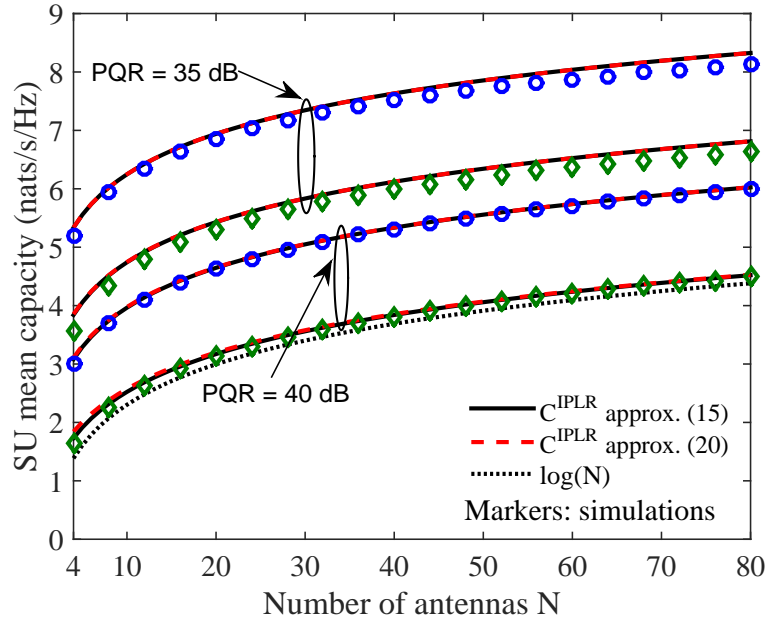


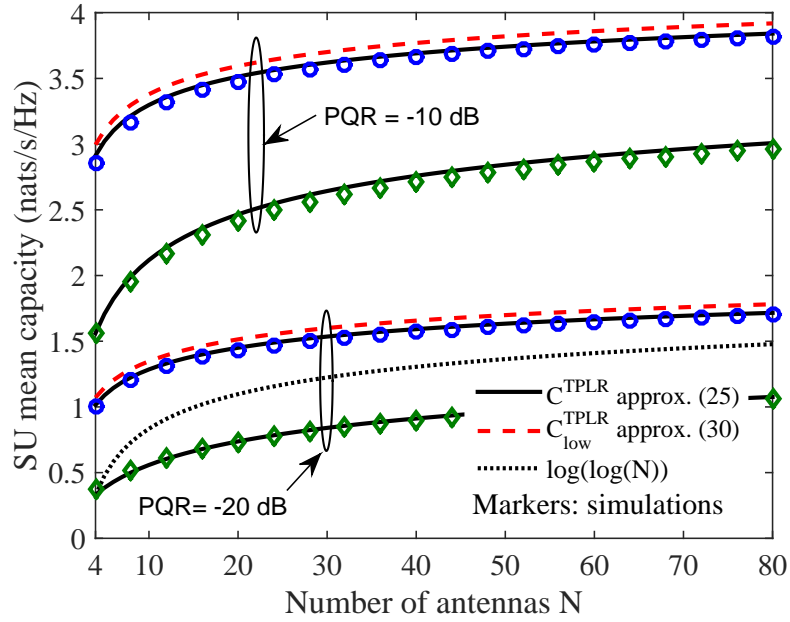
Fig. 2. SU mean capacity versus the peak interference power constraint Q with $P_{\max} = 0$ dB and $P_p = 0$ dB.

Fig. 3 shows the the SU mean capacity as a function of the number of ST antennas in the interference power and the transmit power limited regimes, respectively. In Fig. 3a, the maximum transmit power constraint is set to $P_{\max} = 30$ dB $\gg Q$. In this case, we calculate the SU mean capacity using IPLR approximation, namely (15) with the normalizing constant a_N^{IPLR} and b_N^{IPLR} given by (18) and (19), respectively. In addition, we show the SU rate scaling (20) in IPLR. Numerical results show that the approximation and scaling yield good agreement with simulations, especially when $\text{PQR} = 35$ dB and $\text{SNR}_Q = 5$ dB. Furthermore, the scaling behavior of the SU capacity is shown to be $\log(N)$ as predicted by (20). In Fig. 3b, we assume that $\text{PQR} = \{-10, -20\}$ dB. The SU mean capacity is computed using TPLR approximation (25), and the SU rate scaling is obtained via (30). In TPLR with low PU interference, e.g. $\text{INR} = -10$ dB, the SU capacity scales as $\log(\log(N))$. Results show that in TPLR, it is more effective to improve the SU capacity by increasing the transmit power than by using more transmit antennas. However, in IPLR if the SU is subject to a lower interference limit, the mean capacity can be maintained by using more transmit antennas.

The SU outage capacity is plotted as a function of the interference constraint Q in Fig. 4,



(a) IPLR



(b) TPLR

Fig. 3. SU mean capacity versus the number of ST antennas N . Lines with circles denote $\text{INR} = -10$ dB; lines with diamonds denote $\text{INR} = 10$ dB. (a) IPLR: $PQR = \{35, 40\}$ dB. (b) TPLR: $PQR = \{-10, -20\}$ dB.

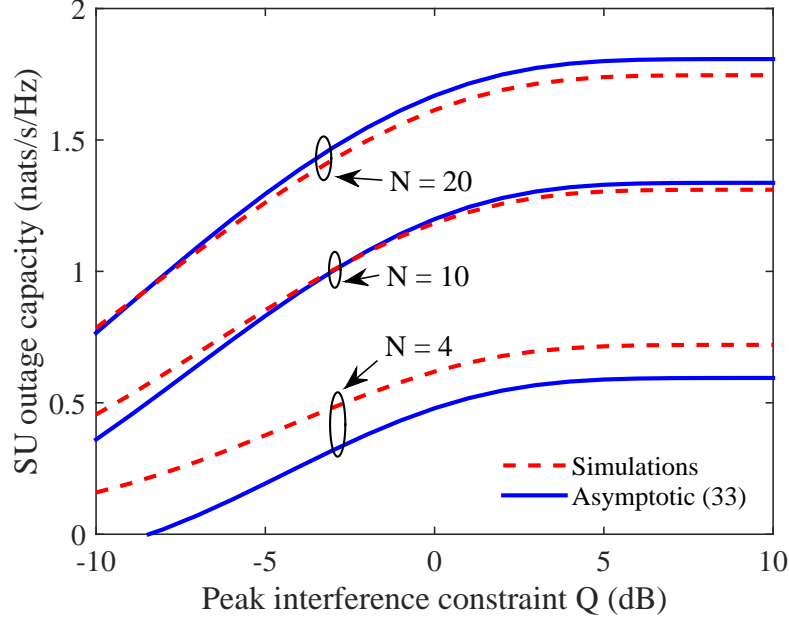
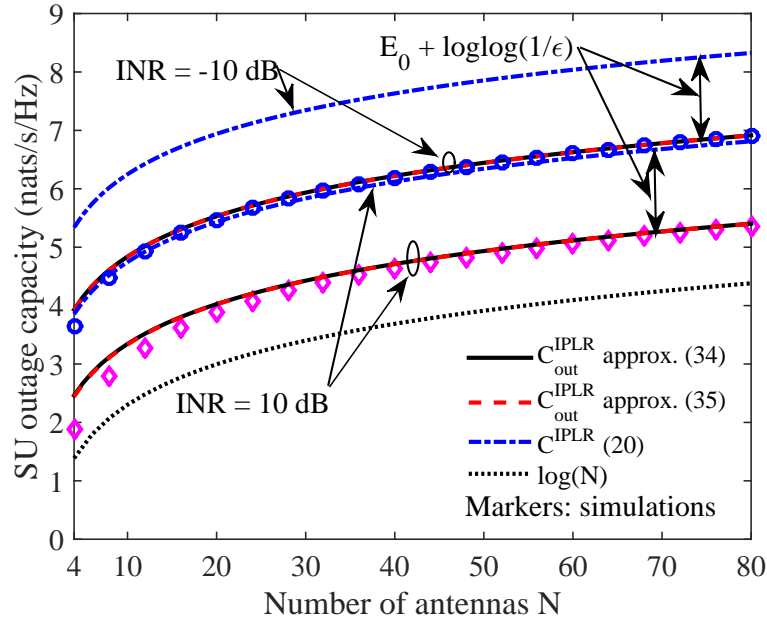


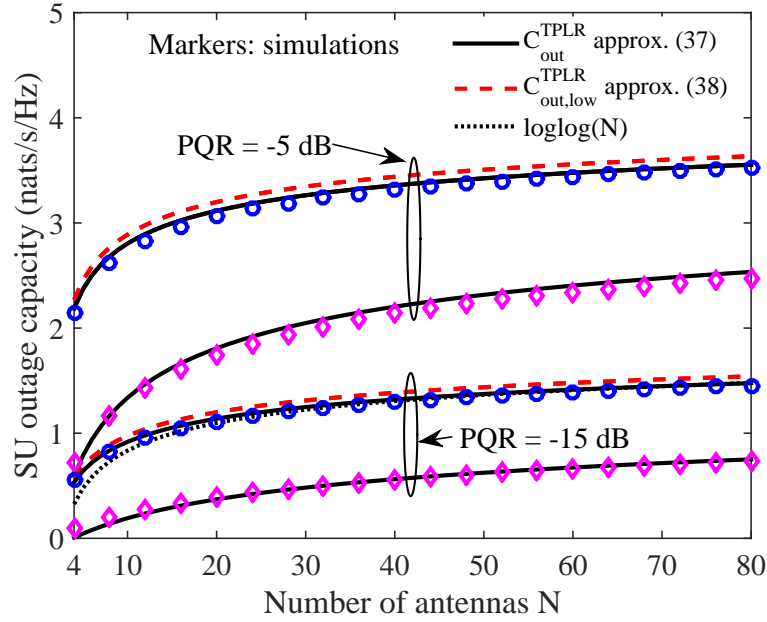
Fig. 4. SU outage capacity. $P_{\max} = 0\text{dB}$, $P_p = 0\text{dB}$, and $\epsilon = 10\%$.

where the outage probability is set to 10%. Although the EVT approximation is not very accurate for $N = 4$ ST antennas, it gives an estimate trend of the SU outage capacity. As the number of antennas increases, the accuracy of the EVT approximation improves. In addition, for $N = 20$ the approximation for a large range interference constraint, $Q \geq -10\text{dB}$, is acceptably accurate for understanding such systems.

In order to gain some insight into the asymptotic behavior of the SU outage capacity, we depict in Fig. 5 the SU outage capacity in the transmit power limited and interference power limited regimes, respectively. The asymptotic SU mean capacity is also plotted for a comparison. As the number of ST antennas increases, the asymptotic outage capacities for various cases are accurate compared to the numerical simulations. The scaling properties of the SU outage capacity are validated. In Fig. 5a, the SU outage capacity in IPLR behaves as $\log(N)$ when the number of ST antennas N approaches to infinity. In this case, there is a constant gap $E_0 + \log \log(1/\epsilon)$ between the outage and the mean capacities as predicted by (36). Yet, the SU outage capacity in TPLR behaves as $\log \log(N)$ as shown in Fig. 5b, which can be proved using (38).



(a) IPLR



(b) TPLR

Fig. 5. SU outage capacity versus the number of ST antennas N . Lines with circles denote $\text{INR} = -10$ dB; lines with diamonds denote $\text{INR} = 10$ dB. The peak interference power constraint $Q = 5$ dB. Outage threshold $\epsilon = 10\%$. (a) IPLR: $\text{PQR} = 25$ dB. (b) TPLR: $\text{PQR} = \{-5, -15\}$ dB.

VI. CONCLUSION

In this paper, we applied the extreme value theory while studying the performance of a spectrum sharing system in terms of the SU mean capacity, the SU outage capacity, and the rate scaling characteristics. The SU mean capacity and outage capacity have two scaling behaviors with the number of available transmit antennas: in the interference power limited regime (IPLR), capacities scale as $\log(N)$, while they scale as $\log(\log(N))$ in transmit power limited regime (TPLR). These results indicate that the transmit antenna selection technique provides different power gain in considered two scenarios. The obtained accurate approximations provide us a better understanding on the performance of the spectrum-sharing systems with multiple transmit antennas.

APPENDIX

DERIVATION OF PROPOSITION 1

The associated PDF becomes, through taking the derivation to $F_{\gamma_i}(x)$ with respect to x and expanding at $x \rightarrow \infty$,

$$f_{\gamma_i}(x) = e^{-\frac{\sigma^2 x}{P_{\max} \bar{g}}} \left(\frac{\sigma^2 \left(1 - e^{-\frac{Q}{P_{\max} \bar{h}}} \right)}{P_p \bar{q} x} + O\left(\frac{1}{x}\right)^2 \right) \quad (39)$$

where $O(z)^m$ represents a term of order $(z)^m$. The expansion at $x \rightarrow \infty$ of the secondary derivative, $f'_{\gamma_i}(x)$, can be expressed as

$$f'_{\gamma_i}(x) = e^{-\frac{\sigma^2 x}{P_{\max} \bar{g}}} \left(-\frac{\sigma^4 \left(1 - e^{-\frac{Q}{P_{\max} \bar{h}}} \right)}{P_{\max} \bar{g} P_p \bar{q} x} + O\left(\frac{1}{x}\right)^2 \right) \quad (40)$$

It is obvious, from (39) and (40), that $\lim_{x \rightarrow \infty} f_{\gamma_i}(x) = 0$ and $\lim_{x \rightarrow \infty} [-f'_{\gamma_i}(x)] = 0$, respectively. We need to show, after taking the derivative in (10), that

$$\lim_{x \rightarrow \infty} \left[\frac{(1 - F_{\gamma_i}(x)) f'_{\gamma_i}(x)}{f_{\gamma_i}^2(x)} \right] = -1 \quad (41)$$

$F_{\gamma_i}(x)$ is the PDF of γ_i , thus we have

$$\lim_{x \rightarrow \infty} [F_{\gamma_i}(x) - 1] = 0 \quad (42)$$

Also, we have $\lim_{x \rightarrow \infty} [f_{\gamma_i}(x)] = 0$. Therefore we obtain, according to L'Hospital's rule,

$$\begin{aligned} \lim_{x \rightarrow \infty} \left[\frac{1 - F_{\gamma_i}(x)}{f_{\gamma_i}(x)} \right] &= \lim_{x \rightarrow \infty} \left[\frac{(1 - F_{\gamma_i}(x))'}{f_{\gamma_i}'(x)} \right] \\ &= \lim_{x \rightarrow \infty} \left[\frac{f_{\gamma_i}(x)}{-f_{\gamma_i}'(x)} \right] = \lim_{x \rightarrow \infty} \left(\frac{P_{\max} \bar{g}}{\sigma^2} + O\left(\frac{1}{x}\right) \right) = \frac{P_{\max} \bar{g}}{\sigma^2} \end{aligned} \quad (43)$$

Thus, $\lim_{x \rightarrow \infty} \left[\frac{1 - F_{\gamma_i}(x)}{f_{\gamma_i}(x)} \right]$ or $\lim_{x \rightarrow \infty} \left[\frac{f_{\gamma_i}(x)}{-f_{\gamma_i}'(x)} \right]$ both have a finite limit. Therefore, we have

$$\begin{aligned} \lim_{x \rightarrow \infty} \left[\frac{(1 - F_{\gamma_i}(x)) f_{\gamma_i}'(x)}{f_{\gamma_i}^2(x)} \right] \\ = \lim_{x \rightarrow \infty} \left(\frac{1 - F_{\gamma_i}(x)}{f_{\gamma_i}(x)} \right) \lim_{x \rightarrow \infty} \left(\frac{f_{\gamma_i}'(x)}{f_{\gamma_i}(x)} \right) = -1 \end{aligned} \quad (44)$$

This completes the proof.

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